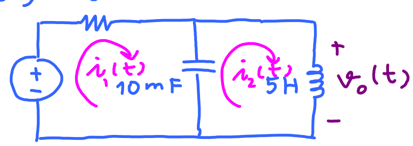
HW12 Q1

Tuesday, April 28, 2015 9:02 AM

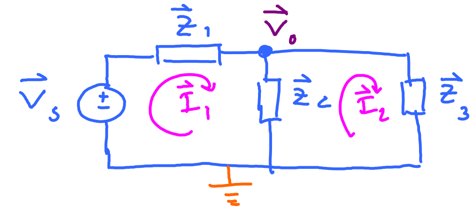
Step 1 : Conversion to phasor form

$v_s(t) = 20\cos(4t - 15^\circ)$

$\omega = 4$



$\Rightarrow$



$\vec{V}_s = 20\angle{-15^\circ} = 19.3 - 5.18j$ V

$\vec{Z}_1 = 60\ \Omega$

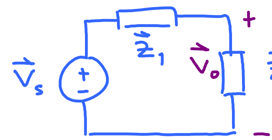
$$\vec{Z}_2 = \frac{1}{j\omega C} = \frac{1}{j \times 4 \times 10 \times 10^{-3}} = \frac{100\cancel{0}}{4j \times 1\cancel{0}} = -25j\ \Omega$$

$$\vec{Z}_3 = j\omega L = j\ 4\times 5 = 20j\ \Omega$$

Step 2 : Solve for the desired variable(s) in phasor domain.

In class, we used impedance combination and the voltage divider technique to find $\vec{V}_o$.

First, we combine $\vec{Z}_2$ and $\vec{Z}_3$ to get $\vec{Z}_{23} = \vec{Z}_2 /\!/ \vec{Z}_3 = \dfrac{1}{\frac{1}{\vec{Z}_2} + \frac{1}{\vec{Z}_3}}$.



Direct Voltage divider

$$\vec{V}_o = \frac{\vec{Z}_{23}}{\vec{Z}_{23} + \vec{Z}_1} \times \vec{V}_s = \frac{1}{1 + \frac{\vec{Z}_1}{\vec{Z}_{23}}} \vec{V}_s = \frac{1}{1 + \frac{\vec{Z}_1}{\vec{Z}_2} + \frac{\vec{Z}_1}{\vec{Z}_3}} V_s$$

$$= \frac{\vec{Z}_2 \vec{Z}_3}{\vec{Z}_2\vec{Z}_3 + \vec{Z}_1\vec{Z}_3 + \vec{Z}_1\vec{Z}_2} \vec{V}_s$$

Note that we keep all values as variables so that we can compare the final answers derived from different approaches easily.

(a) Nodal Analysis :

$$\frac{\vec{V}_o - \vec{V}_s}{\vec{Z}_1} + \frac{\vec{V}_o}{\vec{Z}_2} + \frac{\vec{V}_o}{\vec{Z}_3} = 0 \Rightarrow \vec{V}_o = \frac{\vec{V}_s / \vec{Z}_1}{\frac{1}{\vec{Z}_1} + \frac{1}{\vec{Z}_2} + \frac{1}{\vec{Z}_3}} = \frac{\vec{V}_s}{1 + \frac{\vec{Z}_1}{\vec{Z}_2} + \frac{\vec{Z}_1}{\vec{Z}_3}} = \frac{\vec{Z}_2\vec{Z}_3}{\vec{Z}_2\vec{Z}_3 + \vec{Z}_1\vec{Z}_3 + \vec{Z}_1\vec{Z}_2} \vec{V}_s$$

(same as above)

(b) Mesh Analysis

$\vec{V}_s - \vec{I}_1\vec{Z}_1 - (\vec{I}_1 - \vec{I}_2)\vec{Z}_2 = 0 \leftarrow$ KVL around mesh 1

$-(\vec{I}_2 - \vec{I}_1)\vec{Z}_2 - \vec{I}_2\vec{Z}_3 = 0 \leftarrow$ KVL around mesh 2

$(-\vec{Z}_1 - \vec{Z}_2)\vec{I}_1 + \vec{Z}_2\vec{I}_2 = -\vec{V}_s$

$\vec{Z}_2\vec{I}_1 + (-\vec{Z}_2 - \vec{Z}_3)\vec{I}_2 = 0$

$$\vec{I}_1 = \frac{(\vec{Z}_2 + \vec{Z}_3)\vec{I}_2}{\vec{Z}_2}$$

$$\vec{I}_2 = \frac{-\vec{V}_s\vec{Z}_2}{-(\vec{Z}_1 + \vec{Z}_2)(\vec{Z}_2 + \vec{Z}_3) + \vec{Z}_2^2} = \frac{-\vec{V}_s\vec{Z}_2}{-(\vec{Z}_1\vec{Z}_2 + \cancel{\vec{Z}_2^2} + \vec{Z}_1\vec{Z}_3 + \vec{Z}_2\vec{Z}_3) + \cancel{\vec{Z}_2^2}}$$

$\overline{\qquad\qquad}$
$\vec{Z}_2$

$$-\frac{(Z_1+Z_2)(Z_2+Z_3)}{Z_2} I_2 + Z_2 I_2 = -V_s$$

$$-(\vec{Z}_1\vec{Z}_2 + \cancel{\vec{Z}_2^2} + \vec{Z}_1\vec{Z}_3 + \vec{Z}_2\vec{Z}_3) + \cancel{\vec{Z}_2^2}$$

$$= \frac{+\vec{V}_s \vec{Z}_2}{\vec{Z}_1\vec{Z}_2 + \vec{Z}_1\vec{Z}_3 + \vec{Z}_2\vec{Z}_3}$$

$$\vec{I}_1 = \frac{\vec{V}_s(\vec{Z}_2+\vec{Z}_3)}{\vec{Z}_1\vec{Z}_2 + \vec{Z}_1\vec{Z}_3 + \vec{Z}_2\vec{Z}_3}$$

"Ohm's" law for $\vec{Z}_3$

$$\vec{V}_o = \vec{I}_2 \times \vec{Z}_3 = \frac{\vec{V}_s \vec{Z}_2 \vec{Z}_3}{\vec{Z}_1\vec{Z}_2 + \vec{Z}_1\vec{Z}_3 + \vec{Z}_2\vec{Z}_3}$$

$\leftarrow$ same as above

Plugging in the values of all variables, we have

Step 3: conversion back to time domain

(a) $\vec{V}_o = 16.48 + 4.72j = 17.15\angle 15.6^\circ$ V $\iff$ $v_o(t) = 17.15\cos(4t + 16^\circ)$ V

(b) $\vec{I}_1 = 0.0472 - 0.165j = 0.17\angle -74^\circ$ A $\iff$ $i_1(t) = 0.17\cos(4t - 74^\circ)$ A

$\vec{I}_2 = 0.2359 - 0.8244j = 0.85\angle -74^\circ$ A $\iff$ $i_2(t) = 0.85\cos(4t - 74^\circ)$ A

$\vec{V}_o = 16.48 + 4.72j = 17.15\angle 15.6^\circ$ V $\iff$ $v_o(t) = 17.15\cos(4t + 16^\circ)$ V

Note, it may look strange at first to get -74° for both $\vec{I}_1$ and $\vec{I}_2$.

However, recall that $\vec{I}_1 = \frac{\vec{Z}_2 + \vec{Z}_3}{\vec{Z}_2}\vec{I}_2 = \frac{-25j + 20j}{-25j}\vec{I}_2 = \frac{-5j}{-25j}\vec{I}_2 = \frac{1}{5}\vec{I}_2$.

real number $\Rightarrow$ no phase shift

HW12 Q2a

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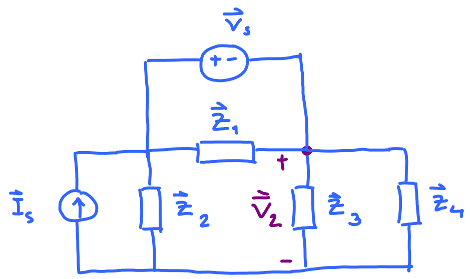
Note that, in this question (and also the examples discussed in lecture), the information is given in phasor domain and the quantities of interest are also the phasor values. Therefore, there is no need to convert back to time domain. In other words, we are focusing on only "step 2".

(a)



$\vec{V}_s = 10 \angle 45°$ V

$\vec{I}_s = 3$ A

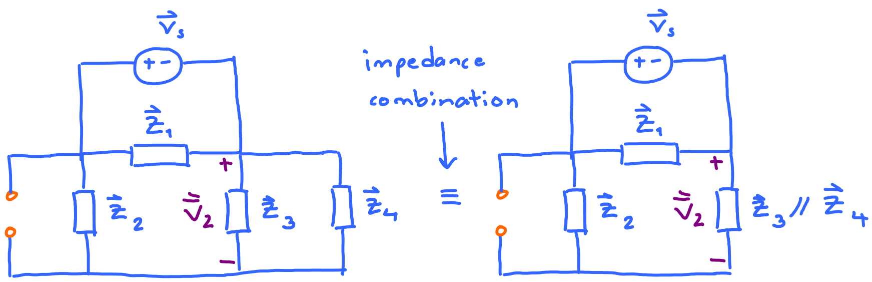
$\vec{Z}_1 = 4\ \Omega$

$\vec{Z}_2 = -3j\ \Omega$

$\vec{Z}_3 = 6j\ \Omega$

$\vec{Z}_4 = 12\ \Omega$

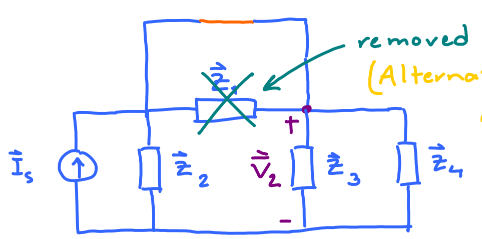
(i) $\vec{V}_s$ acting alone:



$$\vec{V}_2 = -\frac{\vec{Z}_3 // \vec{Z}_4}{\vec{Z}_2 + \vec{Z}_3 // \vec{Z}_4}\vec{V}_s = -\frac{1}{\frac{\vec{Z}_2}{\vec{Z}_3 // \vec{Z}_4} + 1}\vec{V}_s = -\frac{1}{\vec{Z}_2\left(\frac{1}{\vec{Z}_3} + \frac{1}{\vec{Z}_4}\right) + 1}\vec{V}_s = -\frac{1}{\frac{1}{\vec{Z}_2} + \frac{1}{\vec{Z}_3} + \frac{1}{\vec{Z}_4}} \times \frac{\vec{V}_s}{\vec{Z}_2}$$

voltage divider

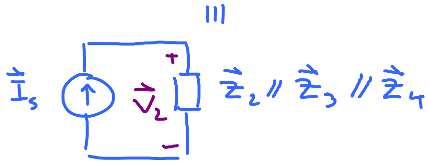
(ii) $\vec{I}_s$ acting alone



removed by the short circuit above

(Alternatively, we observe that there is no voltage across this element. Hence, by "Ohm's law", there won't be any current through it.)

≡



"Ohm's law"

$$\vec{V}_2 = \vec{I}_s \times (\vec{Z}_2 // \vec{Z}_3 // \vec{Z}_4) = \frac{\vec{I}_s}{\frac{1}{\vec{Z}_2} + \frac{1}{\vec{Z}_3} + \frac{1}{\vec{Z}_4}}$$

Now, combine the answers from parts (i) and (ii), we get

$$\vec{V}_2 = \frac{1}{\frac{1}{\vec{Z}_2} + \frac{1}{\vec{Z}_3} + \frac{1}{\vec{Z}_4}} \times \left(\vec{I}_s - \frac{\vec{V}_s}{\vec{Z}_2}\right) = 31.4 \angle -87°$$

same as the expression
derived in lecture via
nodal analysis

HW12 Q2b

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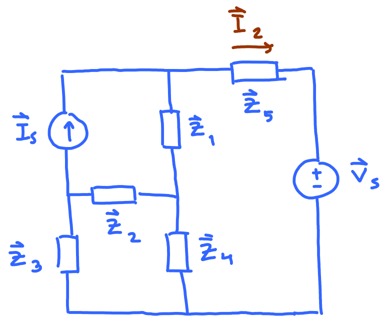
(b)



$\vec{V}_s = 20\angle 90° = 20j$ V

$\vec{I}_s = 5\angle 0° = 5$ A

$\vec{Z}_1 = -2j\ \Omega$

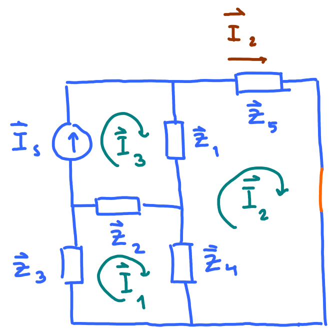
$\vec{Z}_2 = 10j\ \Omega$

$\vec{Z}_3 = 8\ \Omega$

$\vec{Z}_4 = -2j\ \Omega$

$\vec{Z}_5 = 4\ \Omega$

(i) $\vec{I}_s$ acting alone:



Mesh 3: $\vec{I}_3 = \vec{I}_s$

Mesh 1: $-\vec{I}_1\vec{Z}_3 - (\vec{I}_1 - \vec{I}_3)\vec{Z}_2 - (\vec{I}_1 - \vec{I}_2)\vec{Z}_4 = 0$

$$-(\vec{Z}_2+\vec{Z}_3+\vec{Z}_4)\vec{I}_1 + \vec{Z}_4\vec{I}_2 = -\vec{I}_s\vec{Z}_2$$

$$-(8+8j)\vec{I}_1 - 2j\vec{I}_2 = -5\times 10j = -50j$$

$$\Rightarrow \vec{I}_1 = \frac{\cancel{2}j\vec{I}_2 - \overset{25}{\cancel{50}}j}{-\underset{4}{\cancel{8}}(1+j)} = \frac{25j - j\vec{I}_2}{4(1+j)} \quad \ldots (1)$$

Mesh 2: $-(\vec{I}_2 - \vec{I}_1)\vec{Z}_4 - (\vec{I}_2 - \vec{I}_3)\vec{Z}_1 - \vec{I}_2\vec{Z}_5 = 0$

$$\vec{Z}_4\vec{I}_1 - (\vec{Z}_1+\vec{Z}_4+\vec{Z}_5)\vec{I}_2 = -\vec{I}_s\vec{Z}_1$$

$$-2j\vec{I}_1 - (4-4j)\vec{I}_2 = -5\times(-2j) = 10j$$

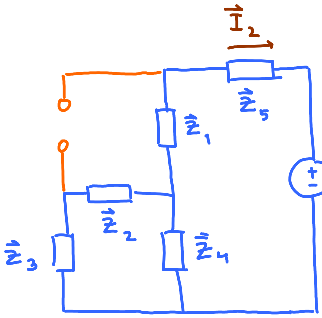
$$\Rightarrow \vec{I}_1 = \frac{\overset{5}{\cancel{10}}j + \overset{2}{\cancel{4}}(1-j)\vec{I}_2}{-\cancel{2}j} = \frac{5j + 2(1-j)\vec{I}_2}{-j} = -5 + 2(1+j)\vec{I}_2 \quad \ldots (2)$$

$$\vec{I}_1 = \vec{I}_1 \overset{(1)\,\&\,(2)}{\Rightarrow} \frac{25j - j\vec{I}_2}{4(1+j)} = -5 + 2(1+j)\vec{I}_2 \Rightarrow 25j - j\vec{I}_2 = -20 - 20j + 8(2j)\vec{I}_2$$

$$\vec{I}_2 = \frac{20+45j}{17j} = \frac{45}{17} - \frac{20}{17}j$$

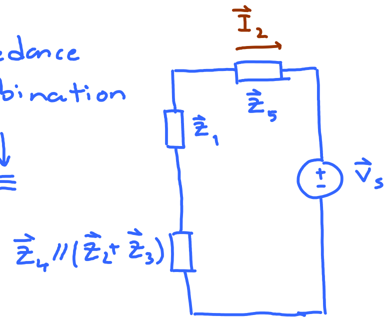
(ii) $\vec{V}_s$ acting alone

(ii) V_s acting alone



impedance combination

$\equiv$



$$\vec{I}_2 = -\frac{\vec{V}_s}{\underbrace{\vec{Z}_1+\vec{Z}_5}_{4-2j} + \underbrace{\vec{Z}_4 // (\underbrace{\vec{Z}_2+\vec{Z}_3}_{8+10j})}} = -\frac{20j\,(2+2j)}{\underbrace{(4-2j)(2+2j)}_{12+4j} + 5-4j} = \frac{40}{17}(1-j)\ A$$

$$\frac{(-2j)(8+10j)}{(-2j)+(8+10j)} = \frac{-\cancel{4}j\,(4+5j)}{\underset{2}{\cancel{8}} + \underset{2}{\cancel{8}}j} = \frac{5-4j}{2+2j}$$

(i) + (ii)

$$\vec{I}_2 = \frac{45-20j}{17} + \frac{40-40j}{17} = \frac{85-60j}{17} = 5 - \frac{60}{17}j = 6.12\angle{-35.2^\circ}\ A$$

[Alexander and Sadiku, 2009, Q11.5]

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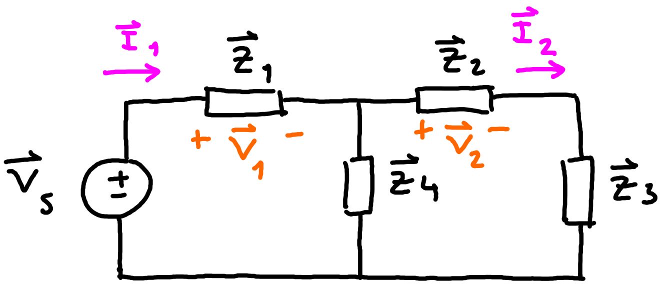
$v_s(t) = 16\cos(2t - 40^\circ)$



Conversion to phasor domain

$\vec{V}_s = 16\ \angle -40^\circ = 12.3 - 10.3j$

$\omega = 2$

$\vec{Z}_1 = R_1 = 1\,\Omega$

$\vec{Z}_2 = R_2 = 2\,\Omega$

$\vec{Z}_3 = \dfrac{1}{j\omega C} = \dfrac{-j}{2 \times \frac{1}{4}} = -2j\ \Omega$

$\vec{Z}_4 = j\omega L = j \times 2 \times 3 = 6j\ \Omega$

$$\vec{I}_1 = \frac{\vec{V}_s}{\vec{Z}_1 + \underbrace{\vec{Z}_4 \,/\!/\, (\vec{Z}_2 + \vec{Z}_3)}_{3.6 - 1.2j}}$$

$= 3.04 - 1.44j = 3.366\ \angle -25.38^\circ\ A$

$P_1 = \frac{1}{2}\,\mathrm{Re}\left\{\vec{I}_1^{\,*}\vec{V}_1\right\} = \frac{1}{2}\,\mathrm{Re}\left\{\vec{I}_1^{\,*}\,\vec{I}_1\,\vec{Z}_1\right\} = \frac{1}{2}|\vec{I}_1|^2\,\mathrm{Re}\{\vec{Z}_1\}$

$= \frac{1}{2}(3.366)^2 \times 1 = 5.664\ W$

$\vec{I}_2 = \vec{I}_1 \times \dfrac{\vec{Z}_4}{\vec{Z}_4 + (\vec{Z}_2 + \vec{Z}_3)} = 4.514 + 0.93j = 4.515\ \angle 1.2^\circ\ A$

$P_2 = \frac{1}{2}|\vec{I}_2|^2\,\mathrm{Re}\{\vec{Z}_2\} = \frac{1}{\cancel{2}} \times (4.515)^2 \times \cancel{2} = 20.39\ W$

The average power for capacitor/inductor is 0.

To summarize,

$P_{1\Omega} = 5.664\ W$ $\quad P_{3H} = 0\ W$

$P_{2\Omega} = 20.39\ W$ $\quad P_{0.25F} = 0\ W$

[F2010] ac power analysis

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(a) $p_1(t) = v_1(t)\, i_1(t)$

In HW 11, we found that $v_1(t) = 6.66 \cos(\underbrace{200t + 21.4^\circ}_{A})$ V

In HW 11, we found that $\vec{I}_1 = 1.11 \angle 21.4^\circ$

$\Updownarrow$

$i_1(t) = 1.11 \cos(\underbrace{200t + 21.4^\circ}_{B})$ A

Therefore $p_1(t) = \frac{1}{2} \times 6.66 \times 1.11 \times \cos(\underbrace{0^\circ}_{A-B}) + \frac{1}{2} \times 6.66 \times 1.11 \times \cos(\underbrace{400t + 42.8^\circ}_{A+B})$

$\approx 3.7 + 3.7 \cos(400t + 42.8^\circ)$ W

(b) Method 1 : Take the constant term from part (a)

$\Rightarrow P_1 \approx 3.7$ W

Method 2: Direct Calculation (using the formula discussed in class)

$P_1 = \frac{1}{2} |\vec{I}_1|^2 \operatorname{Re}\{R_1\} \approx 3.697$ W

(c) $P_L = 0$ W $\leftarrow$ Inductor absorbs no average power.

(d) $P_s = -\frac{1}{2} \operatorname{Re}\{\vec{V}_s \vec{I}_1^*\} = -\frac{1}{2} \operatorname{Re}\{(7\angle 30^\circ)(1.11 \angle 21.4^\circ)^*\} \overset{\text{calculator}}{\approx} -3.842$ W

not satisfy the passive sign convention